Hi all,

This Mathematica worksheet is meant to show you how to set up and solve the SB bet-or-check game described in Section 7.3.2 of Expert Heads Up No Limit with asymmetric distributions. Please see the text for a discussion of why and when this is useful.

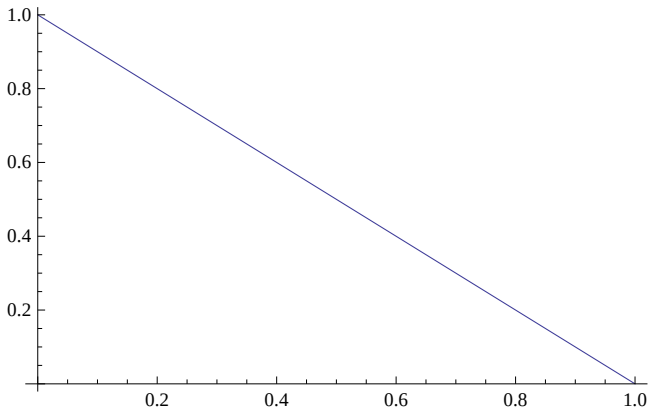
As I mentioned in online Appendix A which works out the dozen equations used to solve the big river game, there's a reason this content has been relegated to the internet rather than included in the book proper. In the book, we focus on developing understanding and intuition rather than just slogging through algebra. This material is provided in the hope that it will help someone, but certainly isn't meant to serve as a preview for EHUNL!

Cheers,
Will Tipton

Ok, let' s start off with the symmetric distributions case. The SB's equity distribution is just a straight line:

```
EQSB[h_] := h

Plot[EQSB[1 - h], {h, 0, 1}, AxesOrigin → {0, 0}]
```



The cutoff hands hc, hb, and hv describe the players' strategies. For each hand, we get one indifference equation. We can write them down with P the size of the pot, B the size of the bet, and S the stack sizes :

```
EVBETHB := (1 - hc) (S - B) + hc (S + P);
EVCHECKHB := S + P EQSB[hb];
EVCALLHC := (S - B) + (P + 2 B) (hb / (hb + (1 - hv)));
EVFOLDHC := S;
EVBETHV := (1 - hc) (S - B + ((EQSB[hv] - hc) / (1 - hc)) (2 B + P)) + hc (S + P) ;
EVCHECKHV := S + P EQSB[hv];

eq1 := EVBETHB == EVCHECKHB;
eq2 := EVFOLDHC == EVCALLHC;
eq3 := EVBETHV == EVCHECKHV ;
```

Let' s choose some specific numbers. Perhaps the SB' s bet is a pot - sized all - in.

```
nums = {P → 1, S → 1, B → 1};
```

With those numbers, we can now solve the three equations to find the three cutoff hands.

```
soln = Solve[{eq1, eq2, eq3} //. nums, {hb, hc, hv}]
```

$$\left\{\left\{\text{hb} \to \frac{1}{9},\ \text{hc} \to \frac{5}{9},\ \text{hv} \to \frac{7}{9}\right\}\right\}$$

The result is as expected. The SB value bets the top 2/9 of his range and bluffs with the bottom 1/9. Thus, the BB' s equity when facing a bet is the same as the pot odds he' s being offered. The BB calls with the top 4/9 of his range. He would have to call a full 1/2 of the time to keep the SB' s air indifferent to bluffing, but here, the SB' s bluffing hands have showdown value.

Now let' s do something a little more interesting -- something that would be a lot harder to do by hand. In the discussion surrounding river Example 1, we explained why the SB bet - or - check decision tree does a good job describing the river play in that hand. Thus, we can solve it to good approximation by solving the above equations just as before except with the river starting distribution from that hand. Please excuse the long listing and just visualize the players' river starting distributions in the plots below.

```
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{0.032061, 0}, {0.030534, 0}, {0.029008, 0}, {0.027481, 0}, {0.025954, 0}, {0.024427, 0},
{0.022901, 0}, {0.021374, 0}, {0.019847, 0}, {0.018321, 0}, {0.016794, 0}, {0.015267, 0},
{0.01374, 0}, {0.012214, 0}, {0.010687, 0}, {0.00916, 0}, {0.007634, 0}, {0.006107, 0},
{0.00458, 0}, {0.003053, 0}, {0.001527, 0}, {0, 0}}, InterpolationOrder → 1];

EQBBfinder[h_] := x /. FindRoot[EQSB[x] == h, {x, 0, 1, 0, 1}];
EQBB =
  Interpolation[Transpose[{Range[0, 1, 0.0001], Table[EQBBfinder[x], {x, 0, 1, 0.0001}]}]];
```
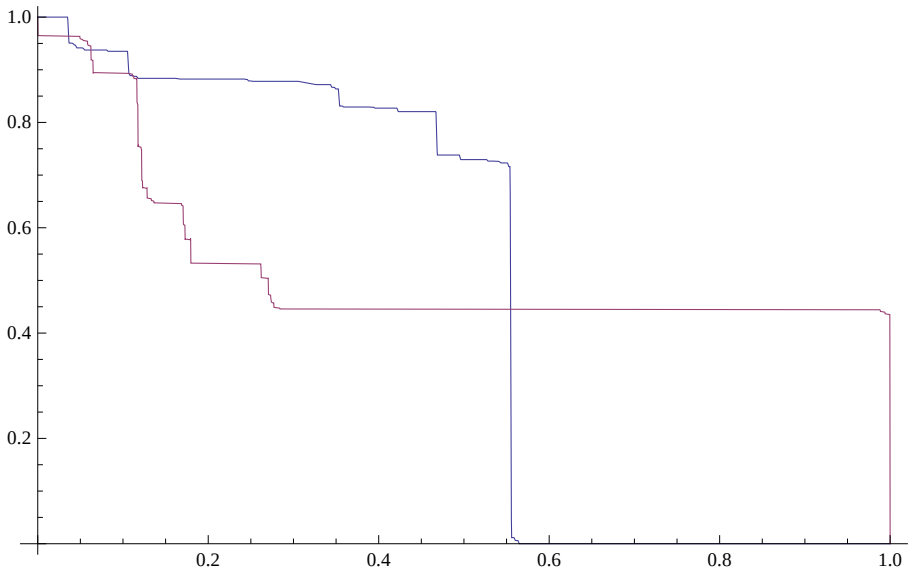

The players' river starting distributions from river Example 1 -- The SB' s distribution is the more - or - less polar one, and the BB' s is the one composed of mostly middling hands :

```
Plot[{EQSB[1 - h], EQBB[1 - h]}, {h, 0, 1}, AxesOrigin → {0, 0}]
```



The relevant sizes from the hand -- the SB is betting all - in with a stack of 17 BB into a pot of 16 BB.

```
nums = {P → 16, B → 17, S → 17};
```

To solve for the cutoff hands in this spot, we' ll use the numerical FindRoot command.

```
soln = FindRoot[{eq1, eq2, eq3} //. nums,
  {{hb, 0.2, 0.3}, {hc, 0.45, 0.55}, {hv, 0.5, 0.55}}, MaxIterations → 10 000]
```

```
{hb → 0.241267, hc → 0.515152, hv → 0.531659}
```

We find that the SB value - shoves almost the top half of his range. As we can see from the equity distribution, this basically corresponds to all of his hands with about 80 percent equity. Check out Example 1 in Chapter 7 for more on this situation.