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Foreword

I've been publishing books for over twenty years and poker books for more than ten. In that time I have commissioned hundreds of books and also received a great many unsolicited manuscripts. As you will probably guess, these have been – shall we say politely – of varying quality. If you've ever watched the *X Factor* auditions, you'll probably get the idea.

I was therefore not unduly excited when a manuscript on heads up no limit hold'em was forwarded to me earlier this year. I have been a serious poker player for ten years but I had never heard of the author. However, he seemed to have already written the whole book – which is unusual in itself. Unsolicited contributions are normally just a few pages or, at most, a chapter. So, I started to read through it.

Within about ten minutes I had experienced what *X Factor* aficionados might call a “Susan Boyle moment”. I was reading one of the very best poker books I had ever seen and I knew absolutely that we would definitely want to publish it. Furthermore I could see that – quite remarkably – the book was already very close to the finished product.

Heads Up No Limit is at the heart of the modern professional game. Will Tipton has taken this variant and stripped it down to the bare essentials – creating a mathematical framework to model all elements of play. It is a quite remarkable book.

Expert Heads Up No-Limit Hold 'em, Vol 1

Poker is a tough game. If you discern a few leaks in your game and want to fix them in five minutes, you'll have to look elsewhere. However, if you are prepared to take a good look at your game, and are willing to work hard to improve, then the information in this book is utterly invaluable.

I've been a solid winning player in the mid-stakes at the online tables for many years. I thought I played okay. Having read this book three times in the last six months I can see that I had huge gaps in my understanding and I am now improving rapidly. If you want to do the same – you should get this book.

Byron Jacobs,
October 2012

Preface

“Serious poker is no more about gambling than rock climbing is about taking risks.” – Alfred Alvarez

Poker, especially the heads-up variety, is about figuring out what your opponents are doing and coming up with a plan to take advantage of it. In this book, we will develop a framework for visualizing an opponent’s strategy and organizing our thought processes to develop exploitative responses. Unlike many other poker books, we will not tell you exactly how you should play particular hands in various situations. We will not give you a fixed strategy for playing heads-up no-limit. This book is designed to teach you to come up with your own strategies for dealing with new situations and opponents as profitably as possible – and to give you the tools to do so.

An approach based on identifying and exploiting opponents’ mistakes is necessary for high-level success in heads-up play because of the nature of the game. With only one opponent to keep track of, you have the opportunity to make many observations about his style and to adjust to it. Your most profitable strategy can and will vary drastically from opponent to opponent. No fixed strategy will be best against all comers. Additionally, even good default strategies in various player pools can change quickly as popular moves and tendencies come and go. So in this book, we will focus

on strategizing as opposed to playing a particular strategy. We will present the math behind adjusting to whatever the future holds.

This book is about both game theory optimal (GTO) and exploitative play, but we will tend to focus on the first of the two. If you are not quite sure what those words mean, don't worry! There is no reason you should, and by the end of Chapter 2, you will be very familiar with the terms. For now, however, GTO play is in some sense "correct". For example, in the game of rock-paper-scissors, the GTO strategy is to throw each choice randomly $1/3$ of the time. Of course, it is hard for humans to be completely random, but insofar as you can, nobody will be able to beat you in RPS in the long term if you play this strategy. However, if you are ever playing a strategy that involves throwing any action with other than a $1/3$ probability, your opponent can take advantage of you. In fact, he can do very well just by figuring out your most likely throw and using whatever counters it 100% of the time, at least until you notice and change your strategy.

This motivates the primary reason we will focus on GTO play. You have to have some idea of what it looks like before you can even start thinking about what your opponent is doing wrong and implementing a strategy to take advantage of it. You must know that the correct rock-throwing frequency is somewhere around $1/3$, before you are able to come to the conclusion that an opponent who throws rock 40% is doing it "too much". Once you know what your opponent is doing and how that deviates from correct play, it is pretty easy to see how to exploit it. The same thing is often the case, to a degree, in poker. Once you know what "correct" play is and can compare it to an opponent's strategy, figuring out an appropriate response is usually not all that difficult. The difference between RPS and poker, however, is that poker is much more complicated. In fact, nobody really knows what this correct play is. As we will see, however, there is a lot that can be done to remedy this situation.

There is a lot of information available about exploitative play (and very little about GTO strategies). However, the notion of "correct" play that is used is generally just based on the author's experience, perhaps with some overly simplified mathematical rules thrown in. A lot of this information is pretty good, but a lot of it is not. Almost all of this advice could work out

well in certain game conditions but be completely disastrous in others. Certainly, this sort of information tends to go out of date as the games change. Game theory optimal strategies, on which the current book is based, will never go out of date. Hopefully this will be a useful text for poker players for a long time.

Although parts of our discussion are moderately technical, we have always tried to keep accessibility in mind. The volume of algebra has been kept to a minimum, and we have used intuitive arguments where possible. Most importantly, applications to real play are always in the foreground. We do not spend any time on the traditional toy games presented in game theory texts – all of our discussion is in terms of genuinely useful heads-up no-limit hold 'em (HUNL) results. The academic prerequisites for understanding the book are fairly minimal: high school level math. Some familiarity with HUNL itself will, of course, also be helpful, although we make an effort to define terms, especially if they have ambiguous meanings in the poker literature.

Rarely in this book will you find examples or discussion which considers the play of a single holding. Doing so encourages and reinforces an incorrect approach to thinking about the game in terms of playing individual hands without reference to the other holdings you might play similarly. As we will see, correct poker strategy involves keeping in mind all of the hands with which you take any of your possible actions. At the beginning, this might be difficult. We have, however, tried to make it fun and to carefully explain more difficult concepts with examples. Read slowly and take time to reflect and apply new ideas to plays you see in your own games.



Also, be sure to use the various “Test Yourself” blurbs placed throughout the text to make sure you understand each new idea before continuing.

We begin in Chapter 1 with a discussion of what it means to talk about poker games and strategies for playing them, and we show how we can describe games and strategies precisely. In Chapter 2, we show how to play most profitably versus opponents whose strategies are known and consider how the situation changes when they are also trying to play well against us. In Chapter 3, we move right into the solution of preflop-only

games. These are some of the simplest HUNL situations, but they apply directly to many real spots in short-stacked play. They will allow us to introduce many important concepts. In Chapter 4, we present the Indifference Principle which will help us to analyze many strategic situations. In Chapter 5, we make a quick discussion of hand and equity distributions which will be our primary tool for visualizing and describing the strengths of players' hand ranges. In Chapter 6, we discuss a general framework for playing hands postflop and establish a context for the rest of the book. In the next few chapters, the first of which is contained in this volume, we discuss GTO and exploitative HUNL play street by street. Future chapters will address the development and implementation of overall game-plans for exploiting opponents, the theory of recursive games which will let us consider the question of when it might be profitable to pass up small edges against weak opponents and "wait for a better spot", and other advanced topics.

I would like to acknowledge some people who have been very helpful to me in my poker career and in the writing of this book in particular. First, I have learned a ton from discussions over the years at the TwoPlusTwo HUSNG online forum and with people I met there. The regular posters there have had a very large influence on my thoughts about this game. I would like to acknowledge posters EvilSteve, AmyIsNo1, and plexiq for providing computational results against which I could check the correctness of my game theory software and for a number of enlightening technical discussions as well. I would also like to thank my students, and especially Yoni "Ph33roX" U., for a steady stream of thought-provoking hand histories and theoretical questions. *The Mathematics of Poker* by Bill Chen and Jerrod Ankenman has had a huge effect on the whole field of practical quantitative poker theory, and it's likely that this book would not have been possible had it not come before – many details of our treatment are derived from their approach. Byron Jacobs, Dan Addelman, Horacio Monteverde and everyone at D&B Publishing have been a pleasure to work with during the production of this book. Finally, my family has been very helpful and understanding during the writing and editing process, and I would like to especially thank Yuki and Michael for their suggestions and edits.

Chapter 1

Introduction to HUNL Strategy

Cards are war, in disguise of a sport. – Charles Lamb

1.1 Heads-up No-limit Hold 'em

No-limit hold 'em play has matured greatly over the past decade due to its increased popularity and acceptance in mainstream culture as well as convenient access to games in online cardrooms.

In traditional brick-and-mortar poker rooms, 9- and 10-handed play was the norm. At the beginning of the poker boom, 6-handed no-limit became many players' game of choice. In these short-handed games, fewer players at the table fought over the same amount of blinds, which meant that the average strength of the winning hand was weaker and that correct play was significantly more loose and aggressive. Players found themselves in more marginal, interesting situations with higher profit potential. This style of play was exciting to many people, and it also allowed skilled players to enjoy larger edges over their opponents.

The benefits of shorter-handed play led to the emergence of 2-handed or heads-up no-limit hold 'em as one of the most popular and profitable

games available. HUNL games are spread in cash, single-table tournament, and multi-table shoot-out formats online and in many live cardrooms as well. The most profitable single-table tournament players online in recent years have been those who play heads-up single-table tournaments (HUSTTs), and many of the most celebrated cash game battles have been heads-up as well.

Since heads-up games pit players *mano-a-mano*, they offer lots of action. As we will see, it is correct to play many hands preflop (often all of them!), and there is no waiting time between hands since any time a player folds, another hand is immediately dealt. The play of so many marginal hands versus a single opponent gives heads-up players the greatest opportunity to observe their opponents' tendencies and to develop effective counter-strategies. In heads-up play more than in any other form of poker, it is important to observe and exploit the tendencies of other players. This is the topic of this book.

Despite the larger skill component present in heads-up games, they are also among the best for beginning players. HUSTTs in particular are well suited to players just starting to learn the game and build their bankrolls. The reason for this is that HUSTTs are spread at stakes as low as \$1 at many popular online cardrooms. For this small buy-in, players receive enough tournament chips to fund dozens of hands of play. Because of the nature of heads-up play, many of these hands are played out postflop. The quick and constant exposure to a large variety of situations is especially valuable to new players.

Small-stakes HUSTTs are also good for building a bankroll since the level of play at the lowest stakes is generally very weak. If players ever do decide to switch to other forms of poker, they often find that the skills they learned heads-up give them a significant edge over other players. Additionally, many new players can be overwhelmed by the need to figure out the play of many opponents simultaneously and are only able to make shallow or cursory reads on their opponents when playing 9- or 6-handed. Heads-up play allows a player to think very deeply about what his opponent is doing, why he is doing it, and how his tendencies can be exploited. This is a very educational experience.

A solid understanding of heads-up play is also important for hold 'em players who never play explicitly heads-up formats. Standard multi-table tournament players are often weak at short-handed and heads-up play for lack of experience. However, the biggest pay increases in these tournaments are in short-handed play at the final table, and the largest is that between first and second place. Thus, it is important that tournament players work on their heads-up games outside of their normal sessions to maximize their expected results.

A short-handed or full-ring cash game player will often want the opportunity to start new tables. This is much easier to do if he is comfortable beginning play with a single opponent instead of a full line-up. Additionally, players' egos do flare at the poker table on occasion, and it is good to have the skills necessary to take advantage of an opportunity to have a weak player all to oneself.

Certain words will be used throughout the book to describe heads-up play. Much of this terminology will be well-known to readers, but we review it for the sake of completeness. Often in analyzing a situation, we will focus on optimizing the play of one player while referring to his opponent's tendencies or strategy. We will call the first player Hero and call his opponent Villain. Try to put yourself in Hero's shoes during these discussions.

Both players must put chips into the pot at the beginning of the hand without seeing any cards – one posts the small blind (SB) and the other posts the big blind (BB), which is exactly twice as much. The abbreviations *SB* and *BB* will be used to refer both to the amounts of chips they contain as well as to the players or positions of the players who post them. Additionally, since the size of these initial bets sets the scale of the action, we will always measure stack sizes in multiples of the BB. Being *on the button* is the same as being in the SB.

Suppose Hero is in the SB. After the blinds are posted, each player is dealt two cards which are known only to him. The SB acts first. He can fold, call, or raise. This first round of betting is known as the *preflop* round. If the preflop action ends with a call, a flop is dealt. The *flop* is three cards which are shared by both players, and we use the word flop to refer both to the cards themselves as well as the round of betting which occurs after they

are dealt. In this second and all subsequent rounds of betting, the BB must act first. After the flop betting, a fourth shared card known as the *turn* is dealt and a third round of betting occurs. Finally, a fifth shared card, the *river*, is dealt and there is a final round of betting. If the river betting is completed without either player folding, the pot is awarded to the player with the best hand made of any five cards from his two private cards and the five public ones. The flop, turn and river are together known as *community cards*.

The smaller of the two players' stack sizes before a hand starts is known as the *effective stack size*. Since no more than this may go into the pot in any single hand, almost all strategic decisions are made with reference to the effective stack. Any additional chips that the larger stack contains are in most cases strategically irrelevant as they can not come into play.

A bet of the entire effective stack is known as an *all-in* bet, and allowable bet sizes in most cardrooms range from one BB to all-in. Allowable re-raise sizes must generally double the size of the previous raise. The smallest re-raise size is known as a minimum raise or a *minraise*.

Preflop, the first raise is called a raise, the first re-raise is called a 3-bet, the second re-raise is called a 4-bet, and so on. During any of the postflop rounds of betting, if a player bets and the other raises, then a subsequent re-raise is called a 3-bet, the next re-raise is a 4-bet, etc.

The final term we will define at this point refers to a very important concept. A *range* is a group of hands, and is often used to describe a player's strategy in a situational context. For example, we may refer to the range of hands which a player raises from the SB preflop. We will often specify ranges of hands using an abbreviated list. For example, the range of hands containing all pairs, all aces, and all suited kings may be written {22+,A2+,K2s+}.

Furthermore, it is possible that a range contains only some fraction of a particular hand. For example, suppose we are facing a raise preflop with A-A and our strategy involves re-raising it half the time and smooth calling the other half. Then, we would say that our re-raising range contains 50% of our A-A hands.

We will see that thinking about hands in terms of players' ranges is fundamental to strategic play. You have to think about your opponent's range to find your own best play. If your opponent is a sophisticated player, he is doing the same thing – he is thinking about your range. Thus, to reason about your opponent's ranges, you have to think about your own. So, for most situations in this book, we will consider the play of ranges versus ranges instead of individual hands.

At certain times in this book, we will refer briefly to the “top $X\%$ of hands.” This is convenient for some examples where the details of the range are not especially important. It does imply some sort of ranking of the hands. There are many different conceivable ways of ranking hands, but for the purposes of these examples, unless otherwise noted, hands are ranked by their chance of winning when all-in preflop versus an opponent with a random hand, that is, an opponent who is equally likely to have any holding. Although hand rankings are sometimes hotly debated, the fact is that there is no single best or most correct method. The relative values of hands always depend on the situation being considered.

More technical terms will be defined as concepts are introduced.

1.2 Games and Strategies

It is often said that the game of heads-up no-limit hold 'em is too large and complicated to be solved with modern computers. However, it might not be clear precisely what is meant by this. What exactly is the entire game of HUNL? How would we write it down? What would it mean to solve it? If we had such a solution, how could we use it to play better poker, anyway? And what strategic progress might be made without the full solution?

We can describe the entire game of HUNL most elegantly by writing it down in the form of a *decision tree*. This tree is often drawn as a set of points connected by lines. Each point, called a *decision point*, represents a spot where one of the players must make a decision to take one or more actions. Each possible action is represented by a line coming off from the

point. We may also refer to the set of actions available to a player as *choices*, *options*, or *strategic options*.

Figure 1.1 is an example of a section of a decision tree. The tree begins at a decision point known as the *root* which we will generally draw at the top. In HUNL games, the root decision point is controlled by the SB, and in this example, he has the option to fold or raise. If he chooses to raise, the play moves down the line labeled “raise” to a decision point controlled by the BB who then has several actions available to him. To fully specify the game, it is also necessary to give the players’ stack sizes at the beginning of the hand.

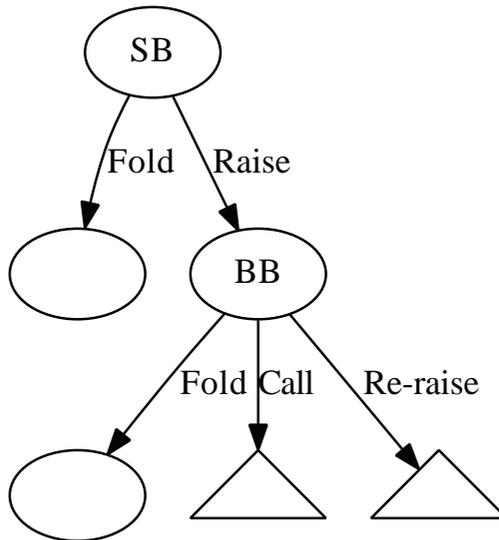


Figure 1.1: Example portion of a decision tree. Triangles are used to indicate that the whole tree continues but is not shown.

This tree segment can be thought of as the beginning of a HUNL hand. After the blinds are posted, the SB acts first, and unless he folds, the BB acts next. Of course, this is a simplified version of a game since, in the full game, the SB has more than two actions open to him at the beginning of the hand. In addition to folding, he may also call or raise to one of many different sizes. Each of these many choices would lead to a different BB decision point.

The BB, similarly, has many choices in the full game. The BB may choose to

fold, in which case the hand is over. He may re-raise, and again each possible re-raise sizing leads to a distinct SB decision point. Additionally, the BB may choose to call. In this case, the game moves to a special type of decision point which is controlled by neither the SB nor the BB. It is generally said to be controlled by God or Nature, and the choice being made is that of which flop to deal. Nature will have similar decisions to make when it comes time to deal the turn and river cards.

Figure 1.2 shows an expanded decision tree which indicates that many raise sizes are possible and that Nature decides which cards to deal at the appropriate points. It also shows a few points which lead to no further action. For example, suppose the SB starts the hand by immediately folding. The hand is over, and we arrive at the point labeled “leaf” on the figure. Since this point is the end of a path through the tree and is furthest from the root, it is known as a *leaf*. (Clever, eh?) In general, these are the decision points reached by a fold, a call of an all-in bet, or a hand-ending action at the end of river play. Since leaves represent the end of a hand, they are where pay-offs happen.

A complete path from the root to a leaf may be called a *line*. This is essentially the same as the use of the word “line” in colloquial poker-speak. It refers to one complete set of actions that both players can take given particular community cards. Additionally, if you start at any particular decision point, call it *A*, then it, along with any decisions and actions which can be reached by moving down the tree are known as the *subtree* or *subgame* beginning at *A*. The two subtrees reached after the SB raises and the BB calls or re-raises are indicated by triangles in Figure 1.1. The *depth* of the tree is the greatest number of actions between the root and any of its leaves.

Now, we have still not shown the decision tree for the full game. Many raise/re-raise/re-raise possibilities have been neglected, not to mention all the possible flops, turns, and rivers. This is in the interest of space. Keep in mind that every possible combination of preflop actions leads to a whole different flop subtree and for each of these, many different flop plays are possible, each of which leads to distinct turn and river subtrees. We will turn to an approximation of the size of the full tree shortly, but first, we consider strategies for playing the games described by a tree.

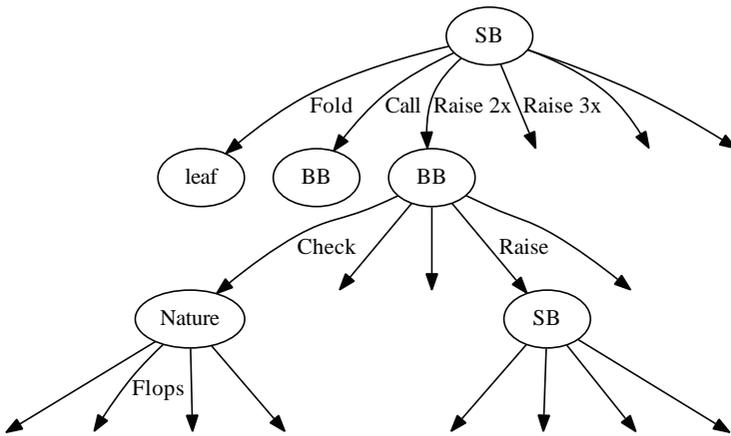


Figure 1.2: Extended decision tree portion.

Now that we know how to describe a game by organizing all of the different situations which may arise into a tree format, how do we describe strategies for playing that game?

Very simply, a player's *strategy* must describe what he will do in any situation he can face in the game. That is, it specifies how he will play each hand in his range at each of his decision points. In practice, this means that to describe a player's strategy, every action which he can take must have a range associated with it which specifies the hands with which he takes that action. Furthermore, at any particular decision point, all of the actions he can take must have ranges that "add up" to the range of hands with which he got to that decision point in the first place.

For example, consider again the simple decision tree segment we looked at in Figure 1.1. At the first decision point, the SB chooses to either raise or fold. Perhaps the range of hands associated with the raise action is any pair, ace, Broadway, or suited hand. The range associated with SB's folding choice must then be all other hands. The two ranges together must contain 100% of hands since the SB has 100% of hands to play at the beginning of the hand.

Now, how big would the full decision tree be if it were to describe all of no-limit? Really Really Big. To find an approximate answer to this question,

we need to count the number of actions possible at each decision point. For example, let us consider again the SB's first action in a hand. He can either fold or call, and we need to specify more about the allowed bet sizes. While there are infinitely many numbers between 0 and any non-negative all-in bet size, in reality, players can not bet any amount. For practical purposes, online players must bet in increments of cents or tournament chips and live players must usually bet in increments of the small blind.

Still, at stack sizes of 100 BB, this only limits us to between several hundred and several thousand possible actions open to the SB at the very first decision point of every hand. That's a lot! For many of those actions, the BB will then have many different actions from which to choose a response. At that point, if there is action remaining after the preflop play, Nature can choose one of the 22,100 flops, and so on.

Keep in mind that each one of those decision points is essentially replicated under each of the similar actions which could lead to it. For example, suppose we had the (huge) subtree that described all the postflop action. That subtree would essentially have to be copied and placed under each one of the 22,100 flops. Then, that combined, gigantic subtree would have to be copied and placed under each of the lines representing all the possible preflop action. And so on. Thus, the size of the tree grows exponentially with the distance from root to leaf – HUNL is a huge game!

The exact size of the decision tree for any given stack size is not important to us, but a quick (under)estimate will help us appreciate the magnitude of the problem. Indeed, if we assume that each decision point gives a choice of 10 different actions, then we can see exactly how the size of the tree grows with its depth. While some decision points, such as those which are the result of a fold, lead to no actions, we have shown that most of them lead to many, many more than 10 actions, so this is a severe underestimation indeed.

A hand which went bet/raise/call on all streets would lead to a distance of about 12 between the root and leaf. Thus, 12 is a reasonable estimate for the average depth of the decision tree. Note that, if stacks are sufficiently deep, many parts of the decision tree will have greater than 12 actions between the root and leaves, and we are again making a rather severe un-

derestimation of the size of the tree. Anyhow, a tree of depth 12 in which each decision point leads to 10 actions has a total of 10^{12} leaves.

Each of these leaves must be the destination of some action which is associated with some range. To specify a range, we must specify what fraction of each of the 1,326 hold 'em hands it contains. If each of these fractions is represented by a single-precision floating-point number, then a single range is about 5KB of data. Thus, a full strategy for HUNL then requires at least $5 \times 10^{12} \text{KB} = 5,000 \text{TB}$ just to store. That is something like the amount of information contained in five billion books of this size, and we have been careful to drastically underestimate the figure at a number of points. The real figure, which has been more carefully estimated elsewhere, is much, much larger. It is significantly more data than any modern computer can process. So, it is hard to say what you would even do if you had the strategy which was some sort of "full solution" to HUNL – it would be too much data to use!

Furthermore, it is obviously not something which could be learned exactly by a human for use at the tables. In this book, we will address these issues in a couple of ways to allow the practical use of game theoretic analysis to improve our play. We will study *approximate games*, those which do not include all of the possible situations in full HUNL. We will find strategies for playing these approximate games and try to extract general patterns and lessons for application to real play.

1.3 Approximate Games

Approximating the full game can make it more manageable from a computational perspective. It also allows us to focus on specific situations in order to distill general lessons from the results without having to take on too much complexity all at once. Therefore, we will often rely in this book on the study of approximate games.

They function as a great tool for analysis in that they allow us to clearly define the strategic options we want to consider and then, within those

assumptions, to analyze the game completely. We will gain important insights by being able to solve realistic HUNL games. When done carefully, ignoring many strategic options does not hurt the analysis. For example, many HUNL players have a single standard raise size they use when first to act preflop on the button at most stack sizes. Even if they mix up this raise size somewhat, very few (winning) players regularly open raise to 5, 6, or 37 times the BB even though they have the option to do so. When we remove options such as these from consideration, the analysis does not lose much.

There are a variety of ways to remove elements from the full game of HUNL while retaining many of its nuances and its rich strategic structure. Some authors organize hands into groups which play similarly and solve for strategies for playing each group instead of each individual hand. For example, A6o-A8o play very similarly and may be considered a single “hand” for many purposes. However, this type of approximation can change proper strategy in subtle ways and make it difficult to connect the results of analysis back to real HUNL.

We will rely on three primary approximation methods:

1) Narrow players’ strategic options

Many lines are always or almost always incorrect, such as putting a large majority of our stack in preflop and then folding. We can immediately exclude these actions from the game. However, we may also neglect lines which are merely uncommon. This is often justified from a strategic standpoint – many plays are just not in a player’s arsenal, so they need not be considered when analyzing his strategy.

Additionally, we will generally consider only a small number of possible bet sizes for a player at any given decision point. If we choose a few sizes which span the range allowed in the full game, we can avoid drastically limiting the players’ strategies. Also, this is similar to how many players actually play, and for good reason. Players regularly bet the same amounts in similar situations for information-hiding purposes.

2) Remove future card possibilities

There are 22,100 distinct flops in hold 'em. After the turn and river are dealt, there are over 2.5 million different boards. Preflop strategy in the full game may be very similar to that in an approximate game with many fewer different flops. Play on the flop may not change much if only a few select turn and river cards can actually come out.

The cards that can come later in a hand, however, do greatly affect play earlier in the hand, so it is important that a variety of different kinds of cards be possible. For example, to obtain preflop strategy results which mimic those in the full game, it is necessary that the approximate game contain about the right number of A-high flops, K-high flops, suited flops, etc. Postflop, it is important that the frequency with which draws come in be kept as close as possible to that in the full game.

In addition to limiting the number of future cards, we might also set up games such that the action is necessarily over on an early street, either by way of an all-in or fold. In this case, we do not really have to consider later cards at all.

3) Consider only part of a hand

Suppose we only want to analyze the river play in a hand. If we assume the stack sizes and ranges with which both players get to the river, we can evaluate river strategies without worrying about the play earlier in the hand. Similarly, we will often assume players' preflop ranges and begin the analysis of a hand after one particular flop.

We will study decision trees and approximate games throughout the book. Our plan will be to start with the simplest approximations and work our way to more complex ones. This will make it easier for us to gain familiarity with our various analytical tools. However, all of the approximate games we consider will be directly related to real HUNL situations. Our goal along the way will be to build intuition about what these solutions look like in order to improve our own play.

In addition to being easier to solve, approximate games are easier to write down and talk about. In fact, computers these days can handle quite large

games – just writing out the decision tree and its solution for one large but computationally-feasible game might take many books. So, we will primarily work with small games and point out results from larger calculations when it is helpful.

There are a variety of ways to represent tree-structured data other than the classical node-link diagrams which we have shown. One format which is convenient for representing trees relatively compactly in print is known as an *outline view*. It is often used to display hierarchical information in computer user interfaces such as file managers. This representation of a tree is conveniently compact. Other presentations such as the *icicle tree* can make use of geometrical areas to clearly indicate how often each action is used. The cover of this book features a *sunburst tree*. The various methods all have their strengths and weaknesses but can be useful for visualizing strategies.

1.4 You Should Now...

- ♠ Know how the game of HUNL is played and why it is an important skill for players of all formats of NLHE
- ♠ Know the meaning of the word “range” and understand the use of the concept in strategic situations
- ♠ Understand how decision trees can be used to model the entire game of HUNL or more specific situations in a particular hand
- ♠ Understand exactly what we mean by a strategy for playing a game described by a decision tree
- ♠ Understand why the simple act of writing down a non-trivial strategy for the full HUNL decision tree would be impossible and have some idea about how we will proceed using approximate games